

Exploring how the geometry of the representation space influences curiosity-based exploration

How does geometry influence exploration behaviors?

In human spatial awareness...

- **3D Projective geometry** structures information integration and planning
- **Subjective perspectives** can be taken on this internal representation space

Methodology

- **Agent** a looks for an **object** O using **observations** $y_o \in Y$.
- Agent's **"internal world model"** space X is a group structured space, where a group acts on it. **We compare spaces structured by Euclidean Geometry vs Projective Geometry.**
- The agent plans its actions m by taking perspectives on its **internal G-space** X , corresponding to the choice of a **group action**.
- **Beliefs** P_X about the position of object $O =$ **probability measure** on the internal space.
- Uncertainty of observations = **Markov Kernel** $P_{Y|X}$. Updated beliefs $P_{X|y_o}$ are computed using Bayes' rule.

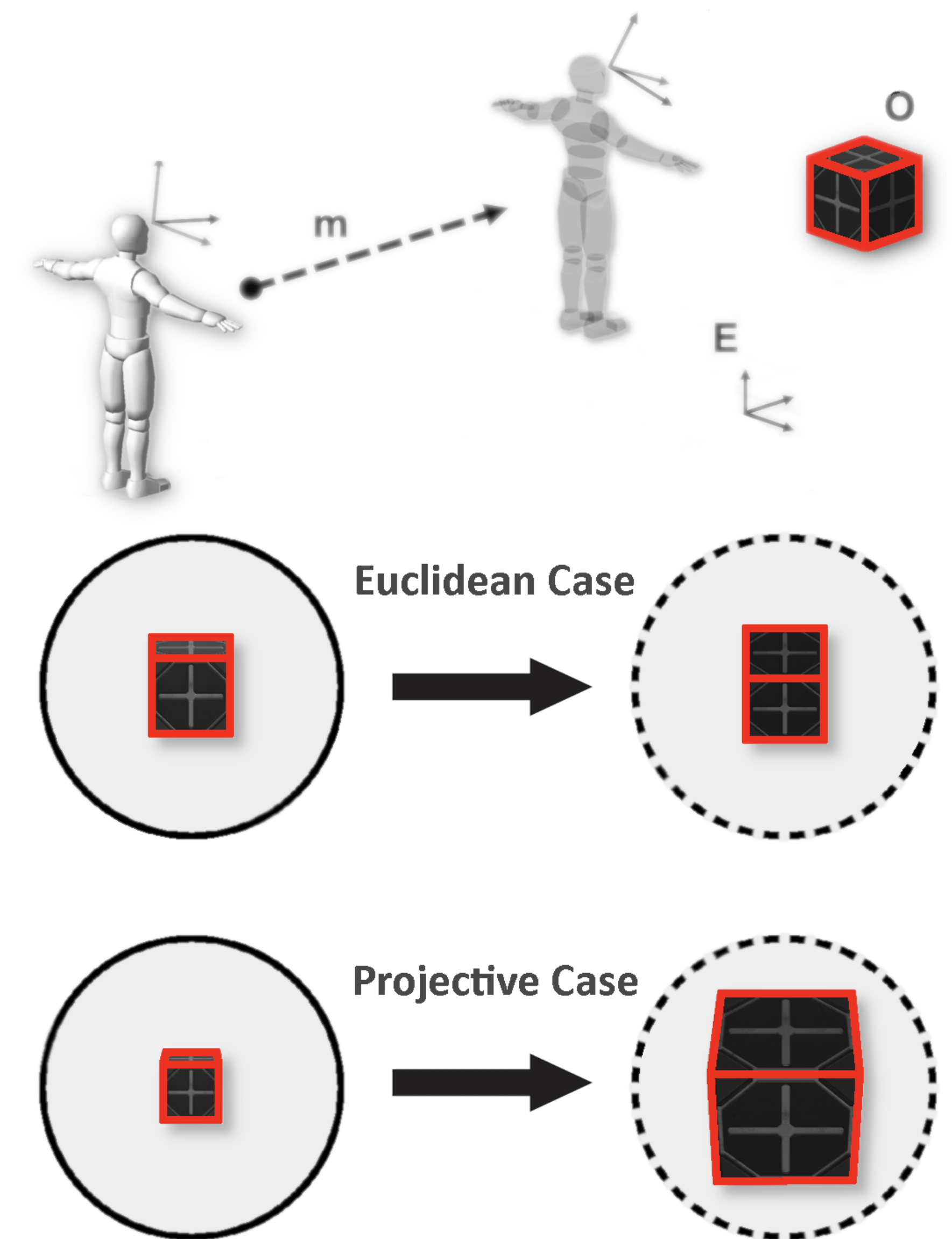
Epistemic value and exploration algorithm

- Epistemic value/curiosity $C =$ **divergence** H **between prior and posterior beliefs** (from Active Inference)

$$C(P_X) := \mathbb{E}_{P_Y} [H(P_{X|Y}|P_X)] \quad (1)$$

- **A broader belief distribution yields a greater epistemic value**
- The agent **maximizes** at each step the epistemic value of the posterior beliefs.

Setup of toy model



Approaching a point in projective geometry **magnifies** the surrounding space.

Results & discussion

I. Geometry can induce a drive to approach the object

Euclidean case : the agent **stays still**

Viewpoint doesn't affect the **broadness of beliefs**: epistemic value is **constant** w.r.t. movement

agent



object



agent



object

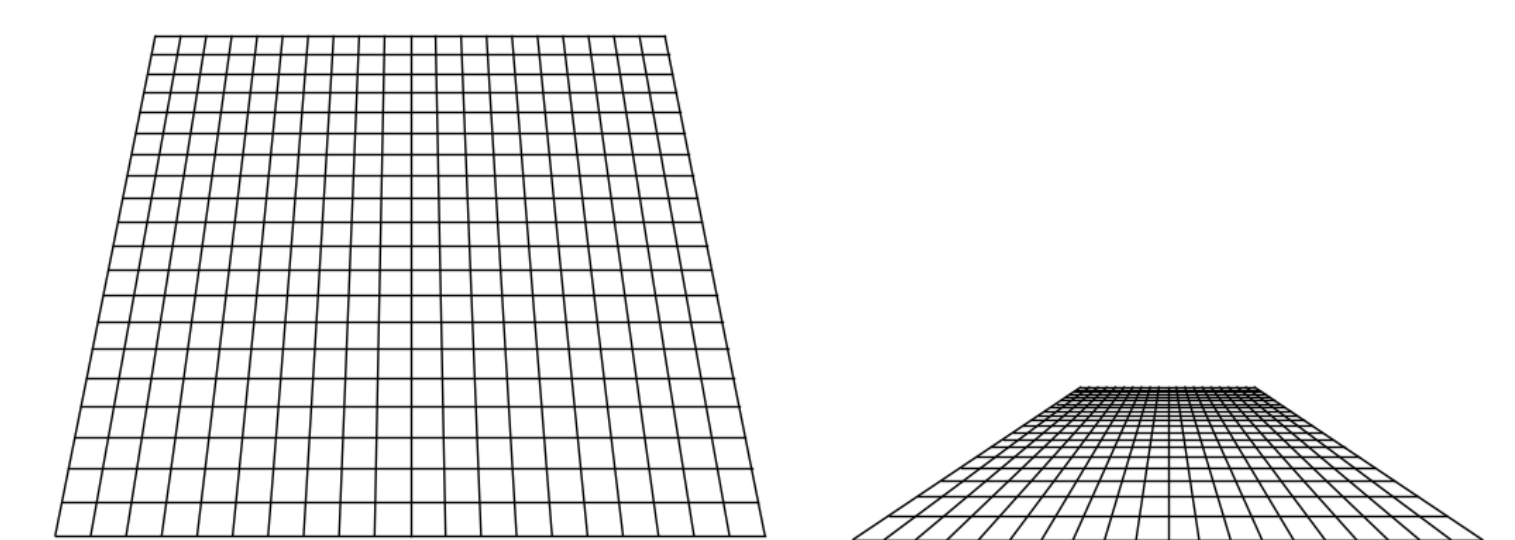


Projective case : the agent **approaches** the object

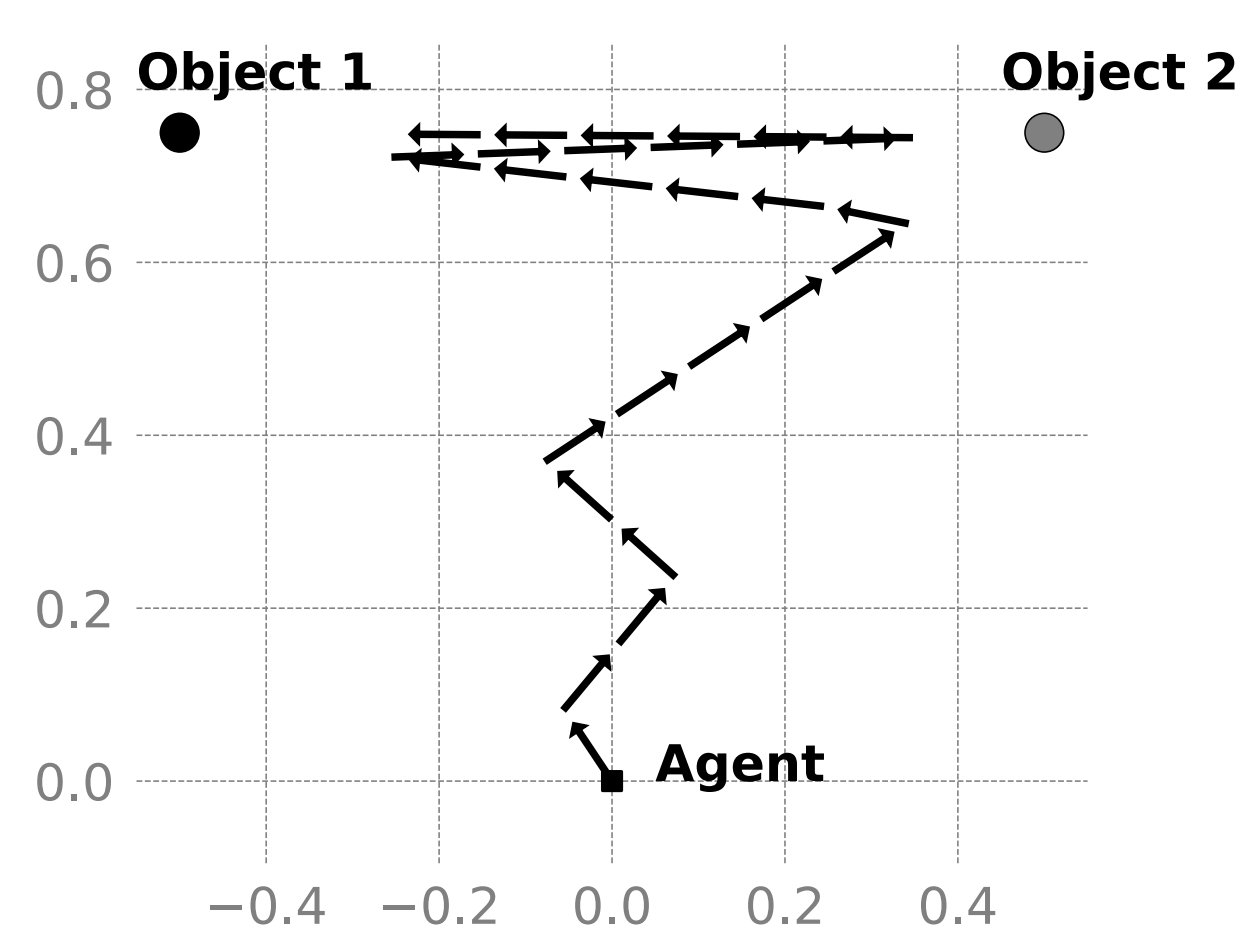
Getting closer **magnifies** the belief distribution = broader beliefs = epistemic value increases as evidence becomes more valuable

II. Exploration behaviors in multi-object projective settings

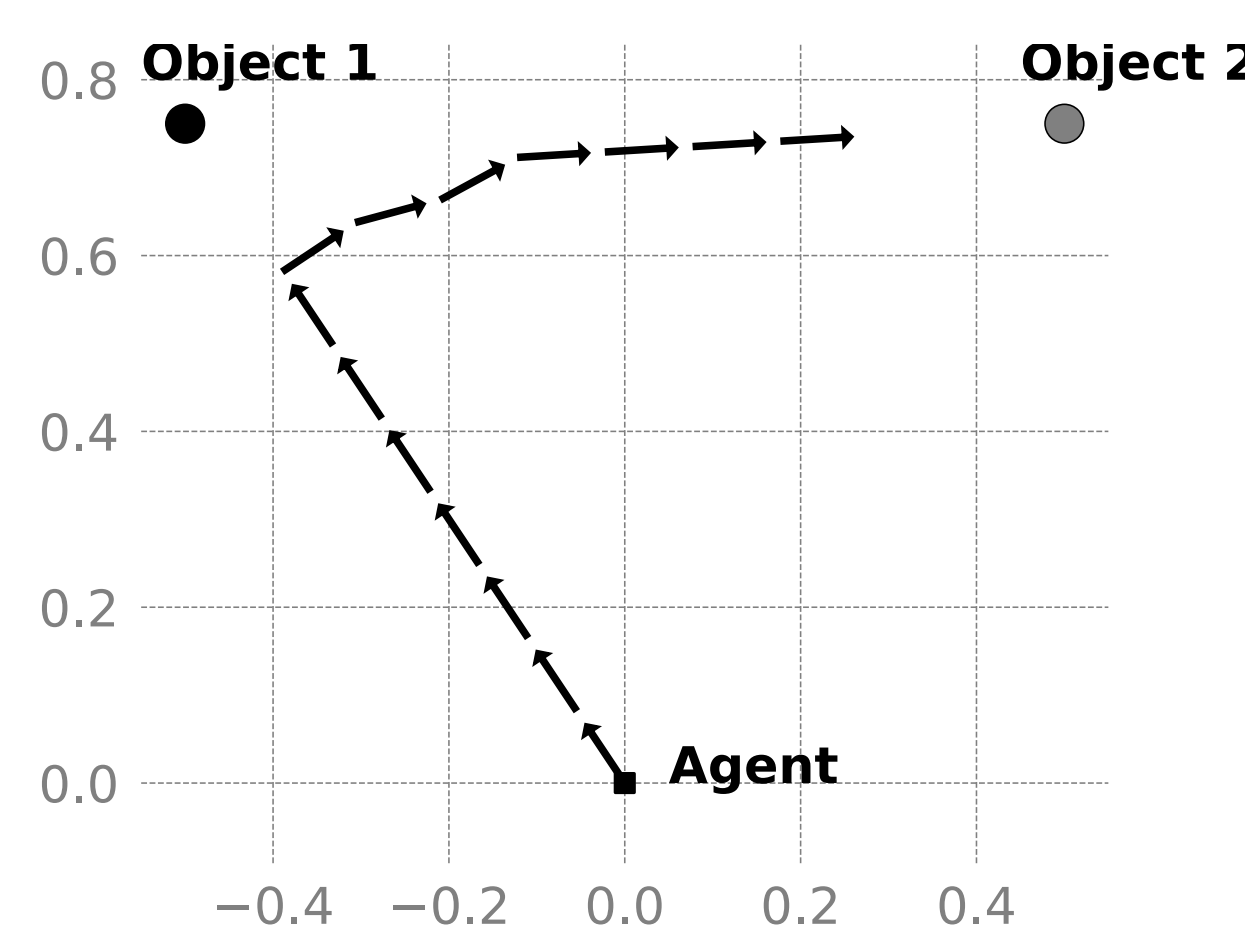
- **Observing** an object **weakens the drive** to approach it as the **epistemic value decreases** (the belief distribution narrows with evidence)
- **Projective geometry** : **approaching** the object **increases the epistemic value** (surrounding space is magnified = broader beliefs)
- Strong projective deformation = **amplified magnifying** = higher drive to approach. This drive may compensate the decrease in epistemic value from the observations.
- Altering the **magnitude of the projective effect** results in **different exploration behaviors**



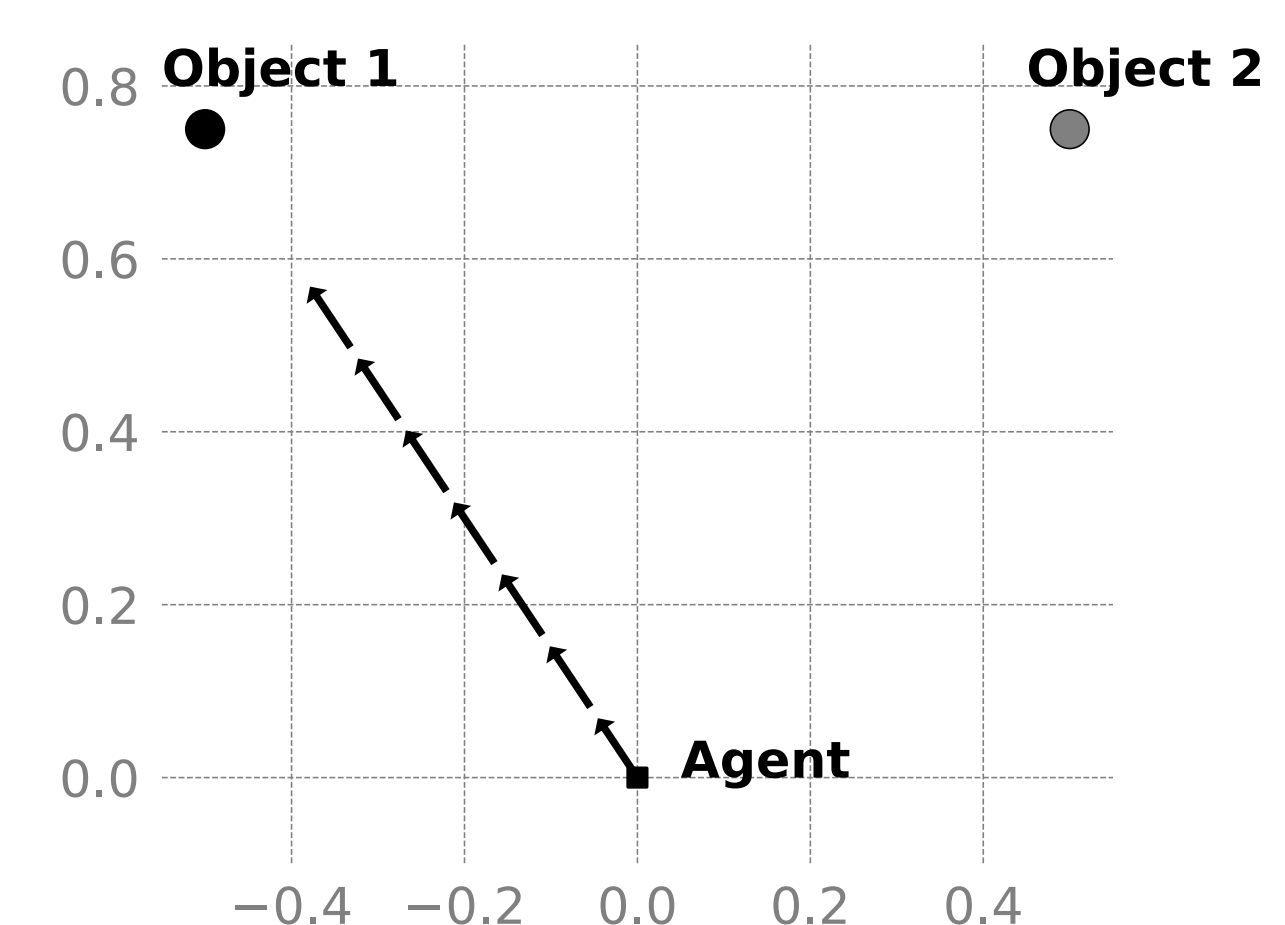
Weak vs strong projective deformation of the euclidean grid. The **magnifying effect** on approach is **amplified with greater deformations**.



Weak projective deformation = lower drive to approach = quick oscillation between objects



Moderate projective deformation = low-frequency oscillation



Strong projective deformation = the drive to approach prevails over the decrease in epistemic value (very low-frequency)

Conclusion

- **Projective geometry** generates an **epistemic drive to approach an object**
- The **magnitude of projective deformations** can be interpreted as **modulating the agent's focus on a target**.

